SOFT MATTER PROBLEMS

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1. **Landau theory for a tricritical point**

   Consider a Landau expansion of the free energy of the form

   $$F = F_0 + \frac{a}{2}m^2 + \frac{b}{4}m^4 + \frac{c}{6}m^6, \qquad c > 0.$$

   Show that there is a line of critical transitions $a = 0, b > 0$ which joins a line of first order transitions $b = -4(ca/3)^{1/2}$ at a tricritical point $a = b = 0$. Sketch the form of the free energy in each region of the (a, b) plane, on the transition lines, and at the tricritical point.

   (For an analysis which includes the terms in odd powers of m see Appendix A of Sarbach and Fisher, Phys. Rev. **B20** 2797 (1979).)

2. **Hedgehogs in a nematic**

   Calculate the elastic energy of a spherical nematic droplet of radius R with a radial hedgehog, in which $\mathbf{n} = (x, y, z)/r$, and with a hyperbolic hedgehog, in which $\mathbf{n} = (-x, -y, z)/r$, assuming no surface anchoring. Show that the radial hedgehog has a lower energy than the hyperbolic hedgehog for $K_3 > 6K_1$.

   (see Lavrentovich and Terent'ev, JETP **64** 1237 (1986).)

3. **Landau theory for liquid crystals + elasticity**

   Consider a liquid crystal in a semi-infinite system bounded at $x = 0$. The director at $x = 0$ is constrained to lie along $\hat{\mathbf{y}}$. A magnetic field is applied along $\hat{\mathbf{z}}$.
   (i) Sketch this geometry and how you expect the director field to behave.
   (ii) Assuming that the director develops no component along $\hat{\mathbf{x}}$ show that the angle θ is makes with $\hat{\mathbf{z}}$ obeys

   $$\xi \frac{d\theta}{dx} = \pm \sin\theta. \qquad (1)$$

   Find an expressions for ξ and interpret it as a length.
   (iii) Integrate Eq. (1) to give

   $$t = e^{-x/\xi}$$

   where $t = \tan\theta/2$.

4. **Active Nematics**

   Read through the active matter lecture notes and complete the three exercises.